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TECHNICAL ARTICLE

Evaluation of Floor Water Inrush based on Fractal Theory and an Improved Analytic Hierarchy Process

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Abstract A new evaluation model, based on fractal theory and an improved analytic hierarchy process (IAHP), was developed to predict the potential for water inrush. Fractal theory was used to quantitatively evaluate the complexity of the fault zones, which is a major water inrush factor. Study of the Lu-an mining area showed that the faults there can be subdivided into four levels of complexity: simple, medium, relatively complex, and complex. The overall complexity of the fault network in the study area was moderate. The IAHP was used to study the potential for coal floor water inrush through these faults. The results indicated that this mining district can be divided into risk-based zones. The extremely high risk zones were mainly located in the northern Tun-liu mine and the northern Chang-cun mine; high risk zones were primarily distributed in the Tun-liu mine and the southwestern Wang-zhuang mine. All other mines were classified as medium and low risk zones.

Keywords Fault · Lu-an mining area · Mine water

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Introduction

From 1998 to 2014, 2433 people died in mine water disasters in China. The deeper the mining, the greater the floor water pressure supported by the coal seam, and the more dangerous the floor water inrush (Liu and Wang 2012; Wang et al. 2011). Therefore, effective forecasting of floor water inrush is necessary for safe coal production.

Many factors affect water inrushes into the mine floor. These factors, which are interrelated, constitute a complex system. However, the conventional water inrush coefficient method (State Administration of Work Safety of China 2009) only considers a few inrush factors, such as the water pressure on the floor aquiclude and the effective aquiclude thickness of the coal floor. It fails to fully describe nonlinear dynamic phenomena, which has led many researchers to look for more reliable methods to assess the risk of mine floor water inrushes. Zhang et al. (2015) studied the mechanisms of water inrush and took measures to prevent it. Wang et al. (2012) assessed water inrush events using to a secondary fuzzy comprehensive evaluation. Wu et al. (2007, 2011a, b, 2013, 2015) have made many outstanding contributions, including establishing a master controlling index system for water bursts. In addition, they used many new methods to evaluate the problem, including the vulnerability index method, a geographic information system (GIS)-based analytical hierarchy process (AHP) vulnerability index model, the AHP approach, and the “three maps-two predictions” method. Li et al. (2008, 2015b) have applied fractal theory to the quantitative study of faults as a basis for assessing the risk of floor water inrush. Wei et al. (2010) and Xiao et al. (2012) evaluated the water inrush risk from coal floors based on fuzzy theory. Dong et al. (2012) set up a GIS-based Bayesian network model to assess water inrush. Li et al. (2015a) used an attribute synthetic evaluation system to assess the risk of floor water inrush.

The advantage of AHP is that the method converts intangible factors into numerical values, and systematically assesses weights to identified causative factors through a series of pairwise comparisons (Saaty 2008). However, AHP requires the calculation of consistency ratios to confirm that judgements are consistent rather than random (Li et al. 2013). Not only does this unnecessarily increase the calculation complexity, but the 10 % recommended threshold for consistency suggests that this criterion is often violated in practice, which calls into question the reliability of the AHP method. The improved analytic hierarchy process (IAHP) introduces an optimal transfer matrix, which can meet the identity requirement (Xie and Xia 2004). This paper applies the IAHP to evaluate floor water inrush risk, after quantitatively evaluating the degree of fault complexity using fractal theory, thus providing a solid technical basis for predicting the risk of floor water inrush.

Using Fractals to Characterize Faults

Water generally floods into mines along faults or shear zones. Therefore, evaluating the complexity of a fault network in a mining area provides a solid foundation for assessing the risk of an inrush occurring. Traditionally, fault density is used to characterize faults quantitatively (Li et al. 2014). However, two areas with the same fault density can have different fault complexity. The fractal dimension of a fracture system is a synthetic index that characterizes the scale, number, fault modularity, and fault dynamic mechanism (Li et al. 2015b). As a result, using fractal dimension instead of single indictors to estimate fault complexity in mines is more reasonable and accurate. This is applicable to both regional- and small-scale faults.

Fractal Dimension

Fractal theory was developed by Mandelbrot (Mandelbrot 1982), who provided a clear interpretation of natural surfaces and other natural phenomena. There are many different definitions of fractal dimension, but in the grid fault, it mainly adopts either the self-similarity or information dimension. The self-similarity dimension D_s (also known as the Hausdorff–Besicovitch dimension) calculation (Li et al. 2013) is:

$$D_s = \frac{\log[N(a)]}{\log(1/a)} \quad (1)$$

where a = geometric similarity ratio and N(a) = the number of similar units within a larger mass. Note that an object is fractal only if D_s exceeds the topological (Euclidean spatial) dimension of the object (Bez and Bertrand 2011; Mandelbrot 1977).

Calculation of the Fault Fractal Dimension

The box-counting method (Fernandez-Martinez and Sanchez-Granero 2014; Feng and Sun 2014) was used to estimate graphical fractal dimensions through a statistical fault trace using a contour map and the following steps:

(1) The study area was divided into numerous planes using longitudinal and latitudinal lines, and these planes were numbered.
(2) We used 1, 4, 16, and 64 smaller planes with side lengths of 1, 1/2, 1/4, and 1/8 of the primary numbered planes, respectively. At different scales, the number N(a) of planes that contain fault traces were counted via a moving-window method.
(3) The slope of the relationship between log(1/a) and log[N(a)] (Eq. 1) was calculated. The value of the slope is the fractal dimension D_s for the fault plane.
(4) The Chinese coal industry standard classification system for geological structures (Ministry of Coal Industry of China 2014) was used to classify the fault network complexity based on the minimum, maximum, and average fractal dimensions. We then gave each grade a different weight value, i.e. if the fault complexity was simple, medium, relatively complex, or complex, its correspondent weight value was 1, 2, 3, or 4.

The Improved Analytic Hierarchy Process (IAHP)

Calculating Theoretical Weight

The specific steps involved in IAHP (Xie and Xia 2004) are:

(1) Build a series of judgment matrices by comparing the importance of control factors in a structured system. For example in matrix A, $a_{ij} = 1$, when “i” is greater than “j”; $a_{ij} = 0$ when “i” is equal to “j”; $a_{ij} = -1$ when “i” is less than “j”.
(2) Calculate the optimal transfer matrixes of the judgment matrixes. If R is the optimal transfer matrix, then:

$$r_{ij} = \frac{1}{n}\sum_{k=1}^{n}\left(a_{ik} - a_{jk}\right) = \frac{1}{n}\sum_{k=1}^{n}\left(a_{jk} + a_{kj}\right). \quad (2)$$

(3) Work out the judgment matrix of the optimal transfer matrix. If D is the optimal transfer matrix of R, $d_{ij} = \exp(r_{ij})$.

(4) Calculate a weight w_i using the following formula

$$W_i = \frac{\sqrt[n]{\prod_{k=1}^{n} d_{ij}}}{\sum_{k=1}^{n}\sqrt[n]{\prod_{k=1}^{n} d_{ik}}} \quad (3)$$

where d_{ij} is the element of the matrix D.

Establishing a Mathematical Evaluation Model

In this paper, we calculated the value of the objective level using the linear weight method. The formula is:

$$Z = \sum_{i=1,j=1}^{n} A_i \times A_{ij} \times A_{ijl} \quad (4)$$

where Z is the value of the objective level; A_i is the theoretical weight of the first layer constraints; A_{ij} is the theoretical weight of the second layer constraints; and A_{ijl} is the actual weight of each factor. If the complexity is complex, relatively complex, medium, or simple, respectively, its corresponding weight value is 4, 3, 2, or 1.

Example Analysis for the No. 15 Coal Seam in the Lu-an Mining Area

The Lu-an mining area (Lu-an area) is located in Changzhi City, China, and includes the Wu-yang, Zhang-cun, Tun-Liu, Wang-zhuang, and Chang-cun mines. The No. 3 and No. 15 coal seams are the ones usually mined in the area (supplemental Fig. 1). As the No. 3 coal resources become exhausted, it is necessary to mine the No. 15 coal seam. However, excavating the No. 15 coal runs the risk of floor water inrush from the Ordovician limestone (OL), which, due to its thickness, water-richness, and high water pressure, threatens mine safety. According to pump test data, the specific discharge of the OL in the study area ranges from 0.28 to 6.94 L/(s·m) (Supplemental Table 1).

The geological structure of the aquifer and aquiclude of the Lu-an area has the following features:

(1) Faults and folds in the Lu-an area are well developed: 33 faults have been found and 15 of them conduct water. Axial lines of some folds develop a lot of fractures, which promote water conductivity. There are 125 documented karst collapses in the study area. The hydraulic conductivity of a karst collapse column is normally high, based on its loose, unconsolidated structure. Hydraulic conductivity is only inhibited where there is good cementation.

(2) The OL lies beneath the No. 15 coal seam and is about 500 m thick. The distance from the top of the OL to the bottom of the No. 15 coal seam is, on average, about 22 m (Supplemental Fig. 1).

(3) The floor aquiclude lies between the OL and the No. 15 coal seam. It consists of the Taiyuan Formation sandy mudstone, Benxi Formation aluminous mudstone (collectively about 22 m thick), and the upper 20 m of the OL (a low water yield zone). The effective thickness of the aquiclude is, on average, about 44 m (Fig. 1).

Analysis of Fault Complexity

The study area was divided into 250 km × 250 km planes using longitudinal and latitudinal lines. The 29 planes that contained a fault trace were numbered from 1 to 29 (supplemental Fig. 2). We then calculated the fractal dimension value in each evaluation using formula (1) (Table 1). Based on the observed conditions for the Lu-an area, we divided the degree of fault network complexity into four levels: simple, medium, relatively complex, and complex (Table 2) by analyzing the maximum, minimum, and average fault fractal dimensions. Based on this estimate, we assigned different grades/weights (if the degree of fault complexity was simple or complex, its relevant weight value was 1 or 4) and drew a partition map of fault complexity (Fig. 1).

Relatively complex and complex faults accounted for 37.5 % of the total (Table 2 and Fig. 1). The average fractal dimension for the faults in the area was 1.218, which indicates that the average fault complexity is moderate (Table 2).

Calculation of the Water Inrush Coefficient

The water bursting coefficient method is a common method for calculating inrush risk that accounts for the pressure from the coal seam and the thickness of the aquiclude. It can be calculated with the formula:

$$T_S = P/M \quad (5)$$

where T_s is the value of water bursting coefficient, in MPa/m; P is the water pressure of the floor aquiclude, in MPa; M is the effective thickness of the aquiclude in the coal seam floor, in m.

Using formula (3), values of Ts were calculated from the values of P and M given in supplemental Table 2 for the

Fig. 1 Zones of similar fault complexity in the study area

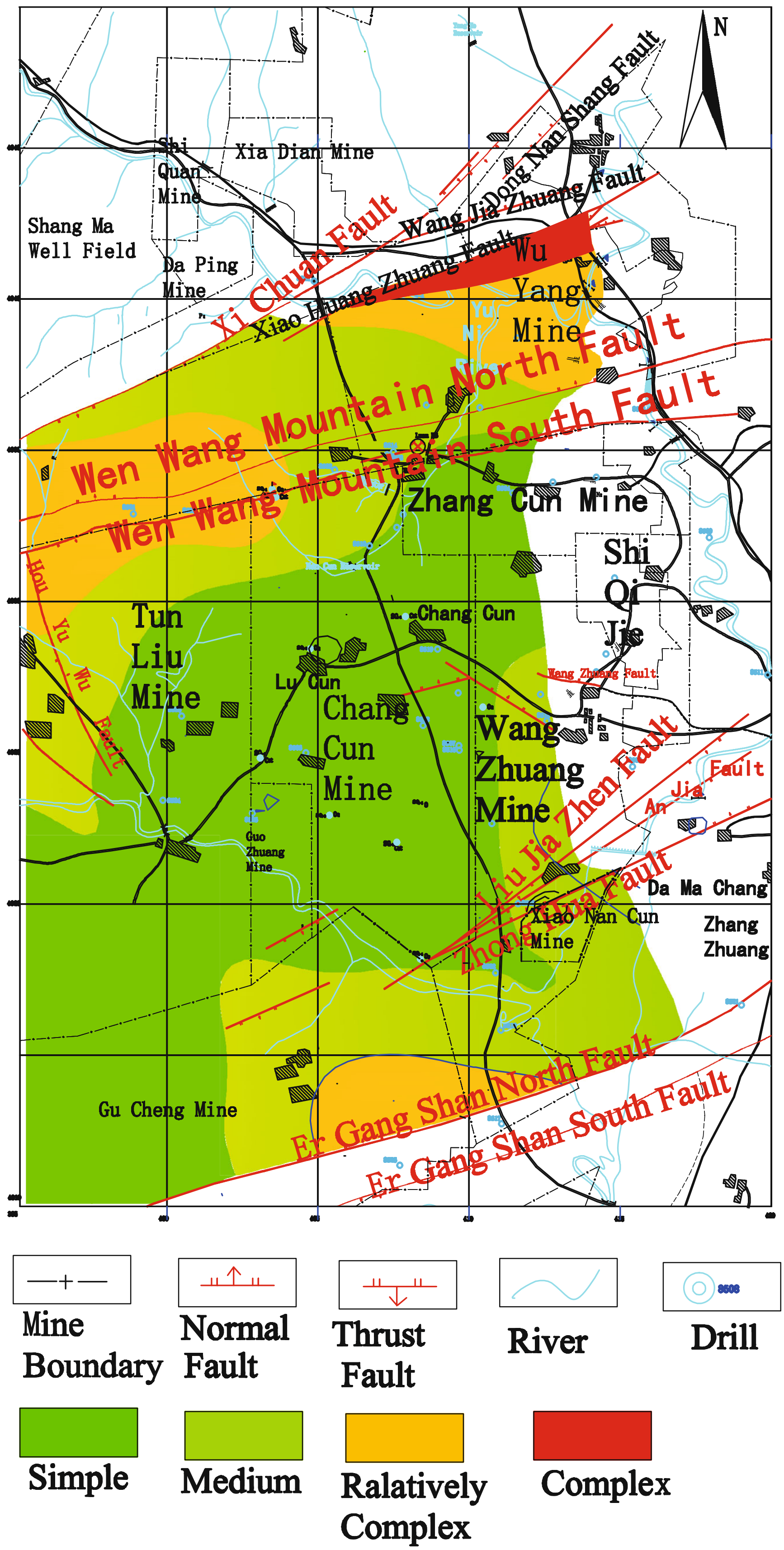


Table 1 Fractal dimension Ds and correlation coefficient R^2 for 29 planes

Plane	D_s	R^2	Plane	Ds	R^2
1	1.600	0.9987	16	0.000	–
2	1.574	0.9919	17	0.000	–
3	1.176	0.9854	18	1.397	0.9977
4	1.232	0.9845	19	1.618	0.9931
5	1.149	0.9821	20	0.000	–
6	1.426	0.9822	21	1.175	0.9854
7	1.414	0.9791	22	1.077	0.9074
8	1.474	0.9847	23	0.959	0.9744
9	1.000	1.0000	24	1.234	0.9911
10	0.300	0.9000	25	0.000	–
11	1.232	0.9845	26	1.129	0.9968
12	0.000		27	1.448	0.9854
13	0.834	0.9902	28	1.470	0.9820
14	1.156	0.9381	29	1.097	0.9949
15	1.051	0.9984	Mean	1.218	–

Lu-an area. The contours of Ts resulting from the values in supplemental Table 2 are shown in Fig. 2.

As Fig. 2 shows, T_s ranged from 0 and 0.187 Mpa/m in the Lu-an area, with values decreasing from west to east. T_s of the Shi-gejie mine was 0 Mpa/m. T_s of Zhang-cun mine was much less than 0.06 Mpa/m. T_s of the eastern Wu-yang mine was generally between 0 and 0.06 Mpa/m, though locally, the western area has a value between 0.06 and 0.14 Mpa/m. The T_s of the Wang-zhuang mine was generally between 0 and 0.06 Mpa/m, while other areas had T_s values greater than 0.06 Mpa/m. The T_s of the Chang-cun mine was generally between 0.06 and 0.14 Mpa/m, though the southern and the northeastern regions were less than 0.06 Mpa/m.

There was a large difference in results calculated for the different mining areas. The water pressure of the OL and the effective thickness of the aquiclude in the coal seam floor vary greatly from one borehole to another in different mines (Supplemental Table 2).

Building a Hierarchal Structure Evaluation System

We established a hierarchal system based on the structural characteristics and features of the area: Level 1 is the water inrush evaluation; Level 2 includes the structural features and the OL aquifer; and Level 3 includes five factors: folding, collapse, fault fractal value, water bursting coefficient and specific yield (see supplemental Fig. 3). We then compared the importance of these factors, based on the actual geological structure and the advice of experts. In the second level of the hierarchy, the presence of the OL aquifer was judged to be more important than structural features. In the third level, the importance of the five factors, from high to low, was judged as: the fault fractal value, folding, collapse, water bursting coefficient, and specific yield. However, the collapse factor was not compared with the water bursting coefficient, since they belong to different upper control factors. Among these five factors, folding, collapse, and fault fractal value are structural features, while the water bursting coefficient and specific yield are based on the OL aquifer. We then built judgment matrices for the control factors based on their relative importance: where $a_{ij} = 1$ when "i" is greater than "j"; $a_{ij} = 0$ when "i" is equal to "j"; $a_{ij} = -1$ when "i" is less than "j".

$$\boldsymbol{A_i} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad \boldsymbol{A_{1j}} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\boldsymbol{A_{2j}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

We then computed the theoretical weight of the control factors based on Eqs. (2), (3), and (4) according to this judicial matrix, and found that the weights of the first level control factors were: $A_i = [0.27, 0.73]^T$; the weights of the second level control factors were: $A_{ij} = [0.29, 0.15, 0.56, 0.73, 0.27]^T$.

Regionalization of Water Inrush from Coal Floor

According to the specific discharge of the OL, we then assessed the groundwater enrichment property of the 29 units that were intersected in the evaluation of fault complexity, following the provisions of the appropriate regulations (State Administration of Work Safety of China 2009). We then assigned them different actual weights (Supplemental Table 3).

We then used formula 3 to calculate the value of the objective level (Z) for each of the 29 units. Using the results, we drew a value contour map for the evaluated regions (Fig. 3). Based on the actual structural situation of Lu-an and previous research (Han and Pan 2013), we

Table 2 Classification of fault networks in the study area

Fault Network Complexity	Simple	Moderate	Relatively complex	Complex
Element weight	1	2	3	4
Fractal dimension range	$D_s < 1.0$	$1.0 \le D_s < 1.3$	$1.3 \le D_s < 1.6$	$D_s \ge 1.6$
Proportion of total (%)	12.5	50.0	29.2	8.3

Fig. 2 Contours of the water inrush coefficient for the study area

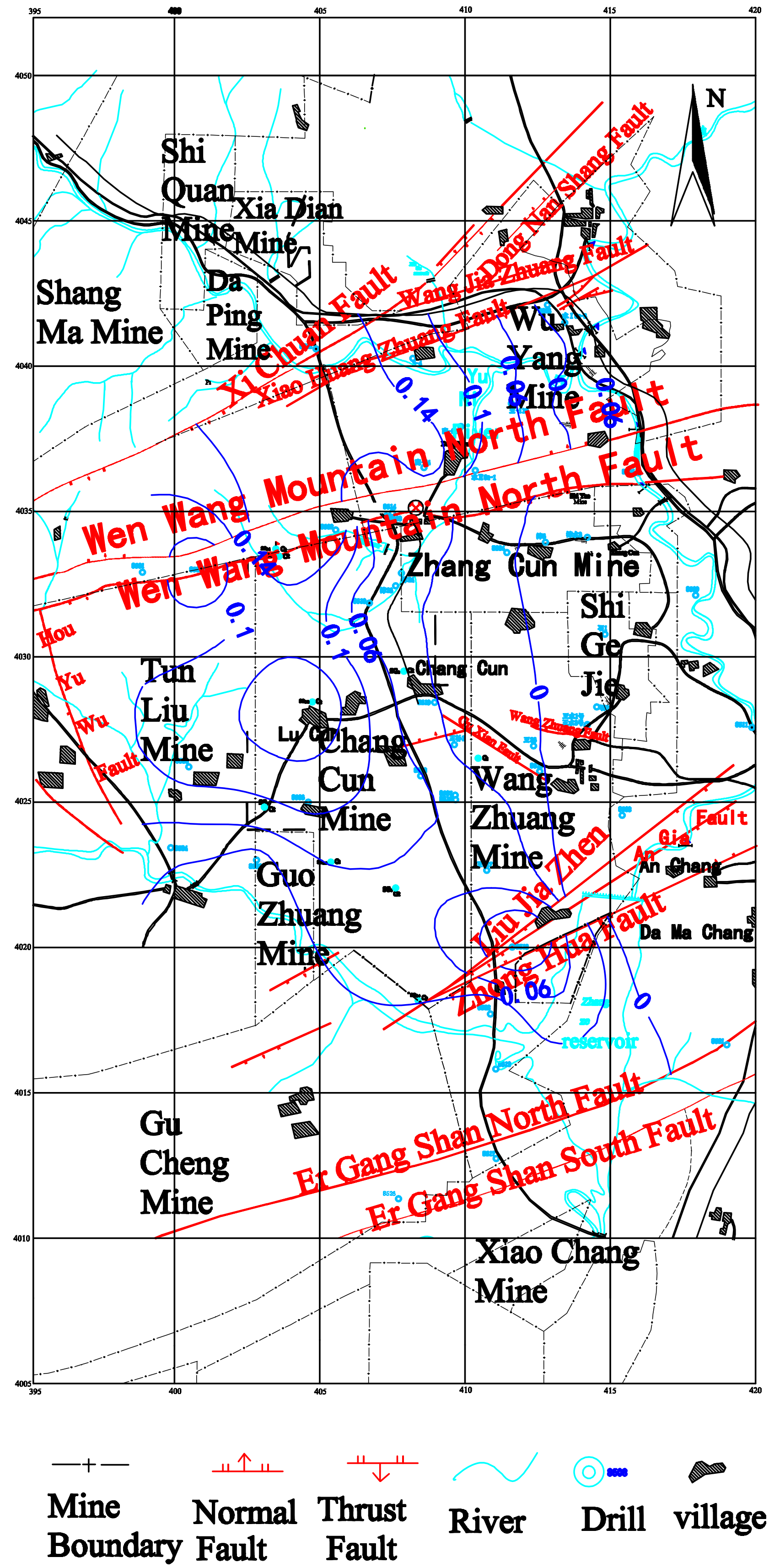
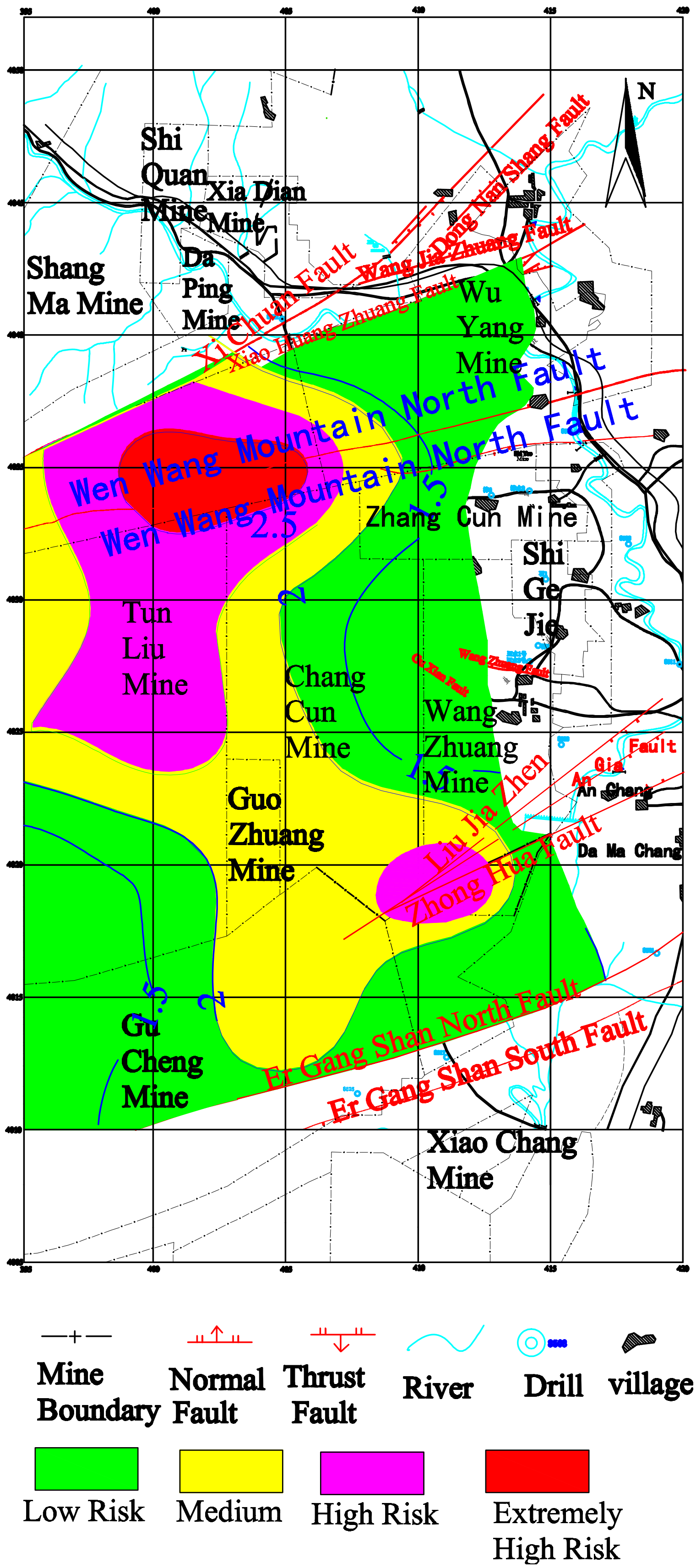


Fig. 3 Evaluation hierarchy for coal floor inrushes



decided that: when Z is less than or equal to 2, it is a low risk area; when Z is between 2 and 2.5, it is a medium risk area; when Z is between 2.5 and 3, it is a high risk area, and when Z is greater than 3, it is an extremely high risk area.

Results and Discussion

Quantitative evaluation using fractal theory for a fault network in a deep mining area in the Lu-an area revealed that the fault fractal dimension averaged 1.218. The fault complexity of the Lu-an area was divided into four different classes: simple, medium, relatively complex, and complex. Among these, simple and medium types accounted for 62.5 %; only 37.5 % were relatively complex and complex. As such, the fault complexity of the Lu-an area is mainly moderate.

Using the IAHP, the water-inrush risk was evaluated for the Lu-an area. The extremely high risk areas were mainly distributed in the northern Chang-cun mine and northern Tun-liu mine. The high risk areas were mainly distributed in the Tun-liu mine and southwest Wang-zhuang mine; the rest are medium and low risk areas.

Using fractal theory to estimate fault complexity not only achieves a quantitative evaluation of the fault but also integrates fault properties, development depth, and fault density into a fractal dimension (*Ds*). Fractal theory can be used for both regional and small scale faults. The IAHP eliminates the need for a conformity test, since it introduces an optimal transfer matrix, which meets the identity requirement, thereby simplifying the calculation. This improves water inrush evaluation objectivity. Compared with the water bursting coefficient method, the IAHP allows a floor water inrush forecast to consider relatively complex and relevant information, including the existence and properties of faults, folds, collapse columns, water bursting coefficients, and specific discharges.

The actual situation of each mine is different. To promote the use of this method, the distribution of karst collapse column data, specific discharge, etc. should be determined. Furthermore, to popularize application of this method in northern China, it is imperative that we build a robust hierarchal structure system to evaluate floor water inrush risk. Of course, during the evaluation, the division of planes will be of great importance. The smaller the grids are, the more accurate the results will be. However, if the grids are too small, the evaluation will be too complex. Therefore, planes should be divided based on the characteristics of the evaluated area.

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